

# An Optimal Resource Allocation with Frequency Reuse in Cellular Networks

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**Abstract**—In this paper, we introduce a novel approach for optimal resource allocation with frequency reuse for users with elastic and inelastic traffic in cellular networks. In our model, we represent users' applications running on different user equipments (UE)s by logarithmic and sigmoid utility functions. We applied utility proportional fairness allocation policy, i.e. the resources are allocated among users with fairness in utility percentage of the application running on each mobile station. Our objective is to allocate the cellular system resources to mobile users optimally from a multi-cell network. In our model, a minimum quality-of-service (QoS) is guaranteed to every user subscribing for the mobile service with priority given to users with real-time applications. We show that the novel resource allocation optimization problem with frequency reuse is convex and therefore the optimal solution is tractable. We present a distributed algorithm to allocate the resources optimally from Mobility Management Entity (MME) to base stations (BS)s sectors. Finally, we present the simulation results for the performance of our rate allocation algorithm.

**Index Terms**—Optimal Resource Allocation, Frequency Reuse, Utility Proportional Fairness

## I. INTRODUCTION

In recent years, the cellular system is witnessing a massive increase in real-time applications. This increase needs to be modeled mathematically for providing efficient resource allocation and scheduling methods. The problem of resource allocation for cellular systems has been under investigation for the last two decade. However, most of the research work conducted focuses on concave utility functions for achieving global optimal solutions. Otherwise, the achieved solution is not optimal. The concave utility functions lack to describe real-time applications as shown in [1]. But the sigmoid utility function provides a very good representation of these applications. However, most of the work conducted on sigmoid utility functions provide sub-optimal or approximate solutions for resource allocation optimization problem, e.g. [2], [3] provides approximate allocation solution and [4] curve fits the sigmoid functions to the closest concave functions to solve with conventional methods. Others only solves for sigmoid utility function above a certain threshold to ensure concavity of the function [5], [6].

In addition, most of the prior work on proportional fairness resource allocation optimization considers proportional fairness over rates [5], [7]. Nowadays, there is a big variety in the applications running under cellular systems operators therefore fairness should be application-based not rate-based. The best representation of application-based fairness is sigmoid utility

for real-time applications and logarithmic utilities for delay-tolerant applications. In [8]–[10], the authors present utility proportional fairness for a cellular system with a single carrier. The authors use logarithmic and sigmoid utility functions to represent delay-tolerant and real-time applications, respectively. In [8], the rate allocation algorithm gives priority to real-time applications over delay-tolerant applications when allocating resources as the utility proportional fairness rate allocation policy is used.

In this paper, we include both sigmoid and concave utility functions for a cellular system with multiple cells. We show how to formulate the resource allocation optimization problem with frequency reuse into a convex optimization problem.

## A. Our Contributions

Our contributions in this paper are summarized as:

- We introduce a novel rate allocation optimization problem with frequency reuse that solves for utility functions that are logarithmic and sigmoid.
- We show that the proposed optimization problem is convex and therefore the global optimal solution is tractable. In addition, we present a distributed rate allocation algorithm to solve it.

The remainder of this paper is organized as follows. Section II presents the problem formulation. Section III shows optimality and duality of proposed problem. In Section IV, we present our distributed rate allocation algorithm with frequency reuse for the utility proportional fairness optimization problem. Section V discusses simulation setup and provides quantitative results along with discussion. Section VI concludes the paper.

## II. PROBLEM FORMULATION

We consider a cellular network model that include cells with sectors. Therefore, we include frequency reuse. In Figure 1, we consider LTE mobile system consisting of  $K$  BSs in  $K$  cells each cell is divided into  $L$  sector (e.g. 3 sectors) and  $M$  UEs distributed in these cells. The rate allocated by the  $l^{th}$  sector BS to  $i^{th}$  UE is given by  $r_i^l$  where  $i = \{1, 2, \dots, M\}$ ,  $l = \{1, 2, \dots, L\}$ . Each UE has its own utility function  $U_i(r_i)$  that corresponds to the type of traffic being handled by the  $i^{th}$  UE. Our objective is to determine the optimal rates that the  $l^{th}$  carrier BS should allocate to the UEs. The utility functions are given by  $U_i(r_i) = U_i(r_i^1 + r_i^2 + \dots + r_i^L)$  where  $\sum_{l=1}^L r_i^l = r_i$

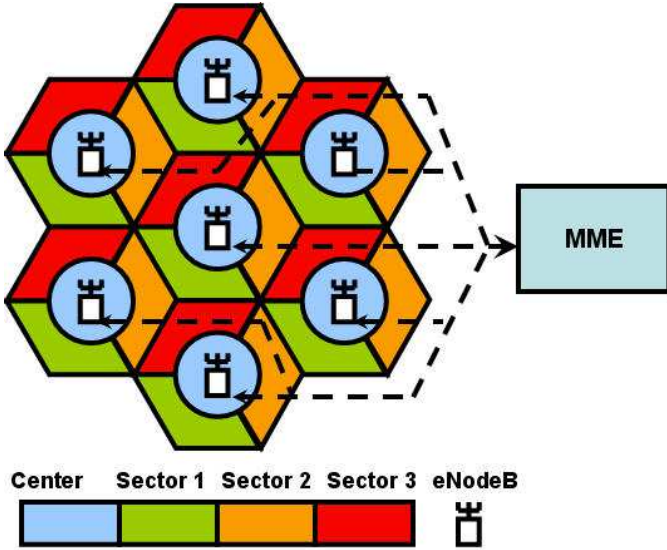


Fig. 1. System Model.

and  $\mathbf{r} = \{r_1, r_2, \dots, r_M\}$ . The optimization problem for utility proportional fairness:

$$\begin{aligned}
 & \max_{\mathbf{r}} \quad \prod_{i=1}^M U_i(r_i^1 + r_i^2 + \dots + r_i^L) \\
 & \text{subject to} \quad \sum_{l=1}^L r_i^l = r_i, \quad \sum_{i=1}^M r_i^1 \leq R^1, \dots \\
 & \quad \dots, \quad \sum_{i=1}^M r_i^L \leq R^L, \quad \sum_{l=1}^L R^l = R, \\
 & \quad r_i^l \geq 0, \quad l = 1, 2, \dots, L, i = 1, 2, \dots, M.
 \end{aligned} \tag{1}$$

where  $R^l$  is the allocated rate by the MME to the  $l^{th}$  sector, and  $R$  is the sum of the allocated rates to all sectors. We assume that the same frequency band is allocated to all sectors therefore avoiding interference. So we have the assumption that  $R^l$  is fixed for all BSs. We assume that UE can't exist in two sectors simultaneously, i.e. if  $r_i^l \neq 0$  then  $r_i^q = 0$  for  $q \neq l$ .

We assume the utility functions  $U_i(r_i^1 + r_i^2 + \dots + r_i^L)$  to be a concave or a sigmoid functions. In our model, we use the sigmoid utility function, as in [3], [11], given by

$$U_i(r_i^1 + r_i^2 + \dots + r_i^L) = c_i \left( \frac{1}{1 + e^{-a_i(\sum_{l=1}^L r_i^l - b_i)}} - d_i \right) \tag{2}$$

where  $c_i = \frac{1+e^{a_i b_i}}{e^{a_i b_i}}$  and  $d_i = \frac{1}{1+e^{a_i b_i}}$ . We use the logarithmic utility function, as in [12], [13], given by

$$U_i(r_i^1 + r_i^2 + \dots + r_i^L) = \frac{\log(1 + k_i \sum_{l=1}^L r_i^l)}{\log(1 + k_i r_{max})} \tag{3}$$

where  $r_{max}$  is the 100% rate and  $k_i$  is the rate of increase..

### III. OPTIMALITY AND DUAL PROBLEM

In the optimization problem (1), since the objective function  $\arg \max_{\mathbf{r}} \prod_{i=1}^M U_i(r_i^1 + r_i^2 + \dots + r_i^L)$  is equivalent to  $\arg \max_{\mathbf{r}} \sum_{i=1}^M \log(U_i(r_i^1 + r_i^2 + \dots + r_i^L))$ , and given that  $\log$  of

sigmoid function is concave as shown in [8] then optimization problem (1) is convex and there exists a tractable global optimal solution.

The primal problem in (1) can be converted into the dual problem following similar steps as in [8]. The UE and BS separate optimization problems follow from similar steps as in [8], [14]. The utility proportional fairness in the objective function of the optimization problem (1) is achieved by the MME which ensure fair allocation to the sectors to achieve equal shadow price for all users. This is done by setting  $p_l = p$  for all  $l$ . We define the aggregated bids of the  $l^{th}$  sectors by  $W^l = \sum_{i=1}^M w_i^l(n)$  and so we have  $W^l = pR^l$  and by summing over  $l$  we have

$$\sum_{l=1}^L W^l = p \sum_{l=1}^L R^l = pR \tag{4}$$

Then it follows from  $W^l = pR^l$  and (4) that

$$R^l(n) = \frac{W^l}{\sum_{l=1}^L W^l} R \tag{5}$$

which is computed in the MME and guarantees fairness.

### IV. DISTRIBUTED OPTIMIZATION ALGORITHM

The distributed resource allocation algorithm is an implementation of UE and BS dual problems, see [8], [14] for more details, and (5). Our algorithm allocates resources from multiple cells simultaneously with utility proportional fairness policy. The algorithm is divided into the  $i^{th}$  UE algorithm shown in Algorithm (1) the  $l^{th}$  sector algorithm shown in Algorithm (2), and MME algorithm shown in Algorithm (3). In Algorithm (1), (2) and (3), the  $i^{th}$  UE starts with an initial bid  $w_i^l(1)$  which is transmitted to the  $l^{th}$  sector. The  $l^{th}$  sector sends the aggregated bids from all UEs under its coverage  $W^l(n) = \sum_{i=1}^M w_i^l(n)$  to MME. MME calculates the difference between the received aggregated bid  $W^l(n)$  and the previously received bid  $W^l(n-1)$  and exits if it is less than a pre-specified threshold  $\delta$  for all sectors. Otherwise, MME calculates the sector rates  $R^l(n) = \frac{W^l(n)}{\sum_{l=1}^L W^l(n)} R$  and sends it to the corresponding sectors. The  $l^{th}$  sector uses  $R^l(n)$  to calculate the shadow price  $p_l(n) = \frac{\sum_{i=1}^M w_i^l(n)}{R^l}$  and sends its value to all the UEs in its coverage area. The  $i^{th}$  UE receives the shadow prices  $p_l$  from all sectors that covers it and calculates the new bid. This process is repeated until  $|W^l(n) - W^l(n-1)|$  is less than the threshold  $\delta$ .

### V. SIMULATION RESULTS

Algorithm (1), (2) and (3) were applied to various logarithmic and sigmoid utility functions with different parameters in MATLAB. The simulation results showed convergence to the global optimal solution. In this section, we present the simulation results of the users placed in Figure 2. The utility function parameters for each user is shown in Table I. In the following simulations, we set  $\delta = 10^{-3}$ .

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**Algorithm 1** The  $i^{th}$  UE Algorithm

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Send initial bid  $w_i(1)$  to BS  
**loop**  
  Receive shadow price  $p(n)$  from BS  
  **if** STOP from BS **then**  
    Calculate allocated rate  $r_i^{opt} = \frac{w_i(n)}{p(n)}$   
  **else**  
    Calculate new bid  $w_i(n) = p(n)r_i(n)$   
    Send new bid  $w_i(n)$  to BS  
  **end if**  
**end loop**

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**Algorithm 2** The  $l^{th}$  sector Algorithm

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Receive sector rate  $R^l(0)$  from MME {Let  $R^l(0) = \frac{R}{L}$ }  
**loop**  
  Receive bids  $w_i^l(n)$  from UEs {Let  $w_i^l(0) = 0 \forall i$ }  
  **if**  $|w_i^l(n) - w_i^l(n-1)| < \delta \forall i$  **then**  
    Allocate rates,  $r_i^{l,opt} = \frac{w_i^l(n)}{p_l(n)}$  to  $i^{th}$  UE  
    STOP  
  **else**  
    Calculate aggregated bids  $W^l(n) = \sum_{i=1}^M w_i^l(n)$   
    Send new aggregated bids  $W^l(n)$  to MME  
    Receive sector rate  $R^l(n)$  from MME  
    Calculate  $p_l(n) = \frac{W^l(n)}{R^l(n)}$   
    Send new shadow price  $p_l(n)$  to all UEs  
  **end if**  
**end loop**

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**A. Rates for  $50 \leq R \leq 1150$** 

In Figure 3, we show the optimal rates of users in the 1<sup>st</sup> sector versus BS rate  $R$ . The optimal resource allocation is content-aware. The users with real-time application (i.e. sigmoid utilities) are allocated resources first. In real-time applications allocation, the user with the steepest utility function (largest  $a$ ) is allocated first as shown in Figure 3.

**B. Bids for  $50 \leq R \leq 1150$** 

In Figure 4, we show the optimal bids of users in the 1<sup>st</sup> sector versus BS rate  $R$ . The users' bids per resource increases

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**Algorithm 3** MME Algorithm

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**loop**  
  Receive bids  $W^l(n)$  from  $l^{th}$  sector {Let  $W^l(0) = 0 \forall l$ }  
  
  **if**  $|W^l(n) - W^l(n-1)| < \delta \forall l$  **then**  
    Allocate sectors rates,  $R^{l,opt} = \frac{W^l(n)R}{\sum_{l=1}^L W^l(n)}$  to  $l^{th}$  sector  
  
    STOP  
  **else**  
    Calculate  $R^l(n) = \frac{W^l(n)}{\sum_{l=1}^L W^l(n)} R$   
    Send new sectors rates  $R^l(n)$  to  $l^{th}$  sector  
  **end if**  
**end loop**

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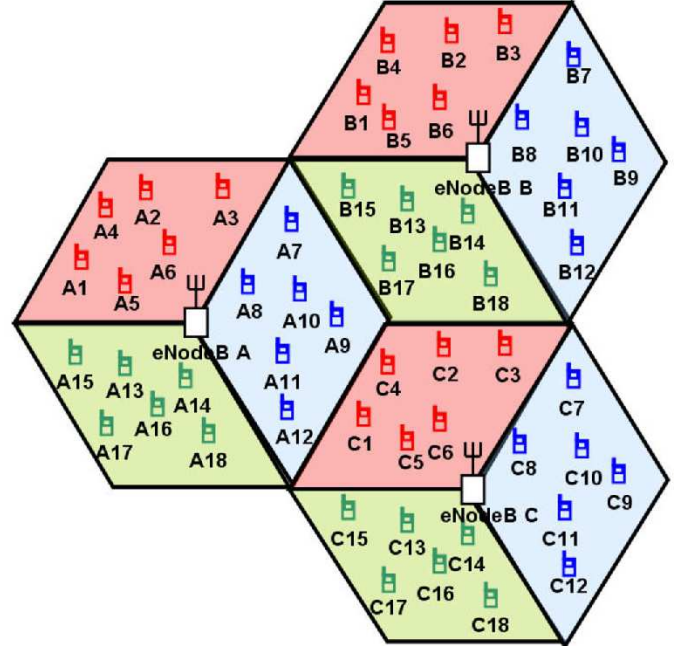


Fig. 2. Simulation setup with three sectors per cell.

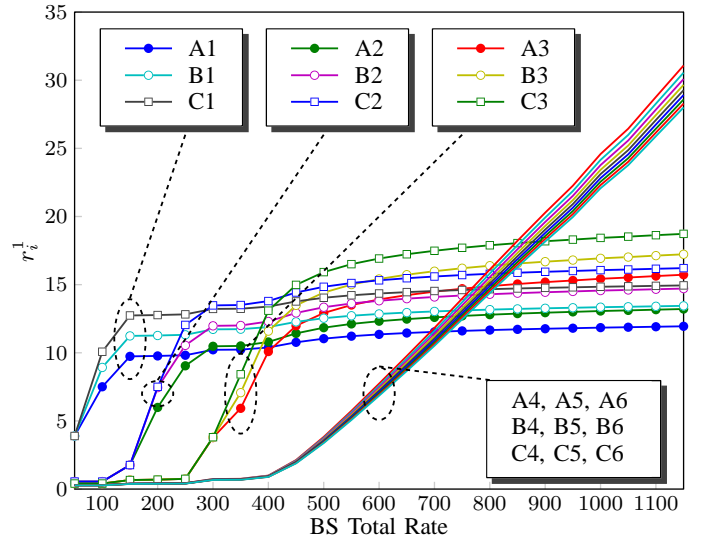


Fig. 3. The rates allocated  $r_i^1$  from sector 1 in the three cells.

as the available resources for allocation in sectors are more scarce, i.e. small values of  $R$ , as the number of users is fixed. Therefore, we have a traffic-dependent pricing. Provided this traffic-dependent pricing, the network providers can flatten the traffic specially during peak hours by setting traffic-dependent bandwidth resource price, which gives an incentive for users to use the network during less traffic hours.

## VI. CONCLUSION

In this paper, we formulated a novel rate allocation optimization problem with frequency reuse. We considered mobile users running two different types of applications, i.e. real-time and delay-tolerant applications, with utility proportional fairness allocation policy in cellular networks. We proved that

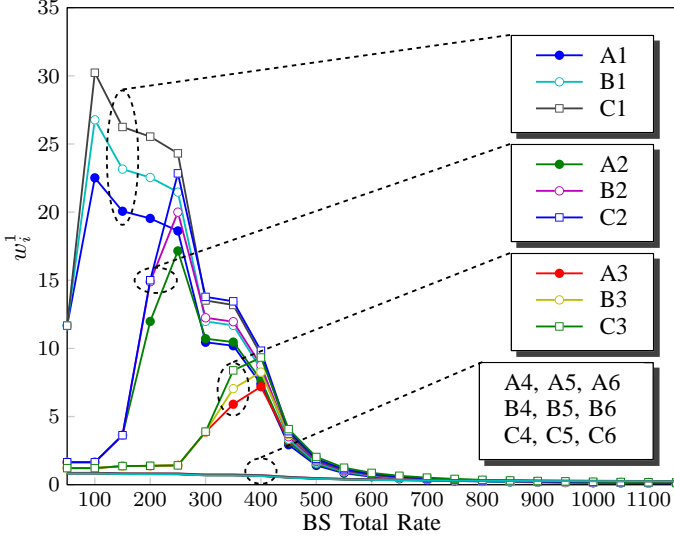


Fig. 4. The bids  $w_i^1$  from sector 1 in the three cells.

TABLE I  
USERS AND THEIR UTILITIES

Sector 1 BS A			
A1	Sig $a = 3, b = 10.0$	A4	Log $k = 1.1, r_{max} = 100$
A2	Sig $a = 3, b = 10.3$	A5	Log $k = 1.2, r_{max} = 100$
A3	Sig $a = 1, b = 10.6$	A6	Log $k = 1.3, r_{max} = 100$
Sector 2 BS A			
A7	Sig $a = 3, b = 10$	A10	Log $k = 1, r_{max} = 100$
A8	Sig $a = 3, b = 11$	A11	Log $k = 2, r_{max} = 100$
A9	Sig $a = 1, b = 12$	A12	Log $k = 3, r_{max} = 100$
Sector 3 BS A			
A13	Sig $a = 3, b = 15.1$	A16	Log $k = 10, r_{max} = 100$
A14	Sig $a = 3, b = 15.3$	A17	Log $k = 11, r_{max} = 100$
A15	Sig $a = 3, b = 15.5$	A18	Log $k = 12, r_{max} = 100$
Sector 1 BS B			
B1	Sig $a = 3, b = 10.9$	B4	Log $k = 1.4, r_{max} = 100$
B2	Sig $a = 3, b = 11.2$	B5	Log $k = 1.5, r_{max} = 100$
B3	Sig $a = 1, b = 11.5$	B6	Log $k = 1.6, r_{max} = 100$
Sector 2 BS B			
B7	Sig $a = 3, b = 13$	B10	Log $k = 4, r_{max} = 100$
B8	Sig $a = 3, b = 14$	B11	Log $k = 5, r_{max} = 100$
B9	Sig $a = 1, b = 15$	B12	Log $k = 6, r_{max} = 100$
Sector 3 BS B			
B13	Sig $a = 3, b = 15.7$	B16	Log $k = 13, r_{max} = 100$
B14	Sig $a = 3, b = 15.9$	B17	Log $k = 14, r_{max} = 100$
B15	Sig $a = 3, b = 17.3$	B18	Log $k = 15, r_{max} = 100$
Sector 1 BS C			
C1	Sig $a = 3, b = 11.8$	C4	Log $k = 1.7, r_{max} = 100$
C2	Sig $a = 3, b = 12.1$	C5	Log $k = 1.8, r_{max} = 100$
C3	Sig $a = 1, b = 12.4$	C6	Log $k = 1.9, r_{max} = 100$
Sector 2 BS C			
C7	Sig $a = 3, b = 16$	C10	Log $k = 7, r_{max} = 100$
C8	Sig $a = 3, b = 17$	C11	Log $k = 8, r_{max} = 100$
C9	Sig $a = 1, b = 18$	C12	Log $k = 9, r_{max} = 100$
Sector 3 BS C			
C13	Sig $a = 3, b = 17.5$	C16	Log $k = 16, r_{max} = 100$
C14	Sig $a = 3, b = 17.7$	C17	Log $k = 17, r_{max} = 100$
C15	Sig $a = 3, b = 17.9$	C18	Log $k = 18, r_{max} = 100$

the global optimal solution exists and is tractable for mobile stations running delay-tolerant and real-time applications. We presented a distributed algorithm, running on UE, BS and MME, for allocating resources to mobile users in different cells and sectors optimally. Our algorithm ensures fairness in utility percentage achieved by the allocated resources for all users. Therefore, the algorithm gives priority to the users with adaptive real-time applications with providing a minimum QoS for all service subscribers. We showed through simulations that our algorithm converges to the optimal rates.

## REFERENCES

- [1] G. Tychogiorgos, A. Gkelias, and K. K. Leung, "Towards a fair non-convex resource allocation in wireless networks," in *PIMRC*, pp. 36–40, 2011.
- [2] G. Tychogiorgos, A. Gkelias, and K. K. Leung, "A new distributed optimization framework for hybrid ad-hoc networks," in *GLOBECOM Workshops*, pp. 293–297, 2011.
- [3] J.-W. Lee, R. R. Mazumdar, and N. B. Shroff, "Downlink power allocation for multi-class wireless systems," *IEEE/ACM Trans. Netw.*, vol. 13, pp. 854–867, Aug. 2005.
- [4] R. L. Kurlle, "Resource Allocation for Smart Phones in 4G LTE Advanced Carrier Aggregation," Master Thesis, Virginia Tech, Nov. 2012.
- [5] E. Björnson and E. Jorswieck, "Optimal resource allocation in coordinated multi-cell systems," *Foundations and Trends in Communications and Information Theory*, vol. 9, no. 23, pp. 113–381, 2012.
- [6] M. Hong and Z. Luo, "Signal processing and optimal resource allocation for the interference channel," *CoRR*, vol. abs/1206.5144, 2012.
- [7] W. Utschick and J. Brehmer, "Monotonic optimization framework for coordinated beamforming in multicell networks," *Signal Processing, IEEE Transactions on*, vol. 60, pp. 1899–1909, April 2012.
- [8] A. Abdel-Hadi and C. Clancy, "A Utility Proportional Fairness Approach for Resource Allocation in 4G-LTE," in *ICNC Workshop CNC*, 2014.
- [9] A. Abdel-Hadi and C. Clancy, "A Robust Optimal Rate Allocation Algorithm and Pricing Policy for Hybrid Traffic in 4G-LTE," in *PIMRC*, 2013.
- [10] A. Abdel-Hadi, C. Clancy, and J. Mitola, "A Resource Allocation Algorithm for Multi-Application Users in 4G-LTE," in *MobiCom Workshop*, 2013.
- [11] M. Ghorbanzadeh, A. Abdelhadi, and C. Clancy, "A utility proportional fairness resource allocation in spectrally radar-coexistent cellular networks," in *Military Communications Conference (MILCOM)*, 2014 *IEEE*, pp. 1498–1503, Oct 2014.
- [12] G. Tychogiorgos, A. Gkelias, and K. K. Leung, "Utility-proportional fairness in wireless networks," in *PIMRC*, pp. 839–844, IEEE, 2012.
- [13] H. Shajaiah, A. Abdelhadi, and C. Clancy, "Multi-application resource allocation with users discrimination in cellular networks," *CoRR*, vol. abs/1406.1818, 2014.
- [14] H. Shajaiah, A. Abdelhadi, and T. C. Clancy, "Robust resource allocation with joint carrier aggregation for multi-carrier cellular networks," *CoRR*, vol. abs/1503.08994, 2015.